Engineering Notes

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Turbulence Modeling in Separated Flow Behind Strong Shocks

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Introduction

TRANSONIC flow over thick airfoils often produces strong shocks with separated flow extending from the shocks to the trailing edge. The governing differential equations most often used to model such flows are the Reynolds-averaged Navier–Stokes equations for a fluid obeying the perfect gas law, Stokes hypothesis for viscosity, and the Fourier heat law. Closure of the Reynolds-averaged Navier–Stokes equations depends on the evaluation of the Reynolds stress terms, which in this Note involves the use of an eddy-viscosity model. Turbulence models using this approach provide good results for attached boundary layers.[1,2] Separated flows, however, provide a much more severe problem and computational procedures that use these turbulence models are often not successful in obtaining stable solutions that match experimental data.

This Note provides a possible explanation of the phenomenon that occurs using an eddy-viscosity turbulence model in shock-induced separated flow. A modification to the Baldwin–Lomax[1] algebraic model that allowed a successful solution for $M = 0.807$ flow over a 16.5% supercritical airfoil (NLR 7301) at 0.37 deg angle of attack is described. The Reynolds number based on chord length was 12×10^6 and the eddy-viscosity was initiated at 10% chord on both the upper and lower surfaces.

The solution procedure used in this study was the MacCormack explicit, unsplit, predictor-corrector method[3] solved on a body-conforming computational grid from a hyperbolic C-grid routine. The 340×85 grid had an initial spacing normal to the surface of 0.0001, a far-field of 15 chord lengths, and Riemann invariant boundary conditions everywhere except the downstream outflow where no change in the streamwise direction was imposed.

Turbulence Model

The Baldwin–Lomax model is a two-layer algebraic model with an intermittency factor. Only the outer layer equation will be discussed here. The equation for the outer eddy viscosity ϵ_o for attached flow over a solid surface is

$$\epsilon_o = \rho K C_{cp} F_{wake} F_{kleb}$$

The variable of interest here is F_{wake} where

$$F_{wake} = Y_{max} F_{max} \tag{1}$$

F_{max} is determined from the equation

$$F(y) = y\,|\omega|\,D$$

by locating the y value at which $F(y)$ is a maximum for a given x location. Y_{max} is the value of y that produces F_{max}.

With a slightly different definition for F_{wake}, the outer eddy viscosity formulation is reported[1] to be valid for separated flow and wakes. The separated-flow/wake formulation for F_{wake} is

$$F_{wake} = (C_{wk} Y_{max} U_{dif}^2)/F_{max} \tag{2}$$

where C_{wk} is a constant and

$$U_{dif} = (u^2 + v^2)_{max}^{1/2}$$

for separated flow over a solid surface. The recommended procedure is to calculate F_{wake} both ways [Eqs. (1) and (2)] and use the smallest value.

Analysis of Results

Using this turbulence model in the solution procedure described resulted in an oscillatory "solution." The oscillations appear to result from the boundary layer being attached on one surface and separated from the other, then reversing. A similar periodic phenomenon has been observed experimentally on an 18% thick circular arc airfoil;[4] however, no such oscillations were observed during the test of this airfoil.[5]

A more detailed investigation of the key parameters in the Baldwin–Lomax model was initiated. In the attached-flow formulation [Eq. (1)], F_{wake} outside of the inner layer is essentially a function of Y_{max} and ω.

$$F_{wake} \sim Y_{max}^2 \,|\omega|_{Y_{max}}$$

When the boundary layer, is thin, Y_{max} is small, and therefore, F_{wake} is small. In separated flow behind a shock, however, the boundary layer thickens rapidly, Y_{max} gets large, and F_{wake} increases as the square of Y_{max}. The resulting large values of eddy viscosity produce large amounts of dissipation in the separated flow region and tends to reduce the extent of separation.

The separated-flow/wake formulation for F_{wake}, [Eq. (2)], outside of the inner layer is essentially

$$F_{wake} \sim C_{wk} U_{dif}^2 / |\omega|_{Y_{max}}, \qquad C_{wk} = \text{const}$$

Notice that boundary-layer thickness does not appear in the expression. Rather, F_{wake} varies as the square of the maximum velocity at the x location of interest. U_{dif} decreases rapidly through the shock, and since F_{wake} is a function of the square of U_{dif}, changes in velocity would tend to have a large and immediate effect on F_{wake}, and, thus, eddy viscosity. The

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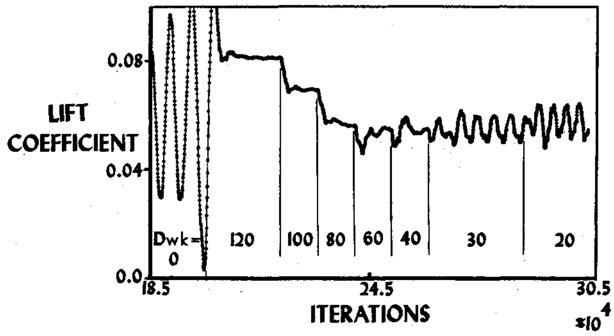

Fig. 1 Effects of weighting parameter on lift coefficient.

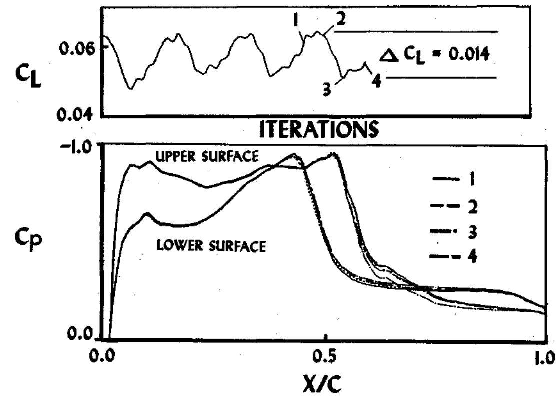

Fig. 2 Sensitivity of pressure data to small oscillations in lift.

Baldwin–Lomax procedure will tend to switch from using Eq. (1) upstream of the shock to Eq. (2) downstream of the shock.

Consider the following hypothesis: Some perturbation causes the shock wave on one surface to become slightly stronger, which reduces the velocity downstream of the shock. Since F_{wake} is proportional to the square of U_{dif}, the immediate effect is that eddy viscosity is reduced. The resulting decrease in dissipation aft of the shock produces a larger separated flow region, drives the shock farther forward, and it becomes still stronger. This produces an even lower U_{dif}, and the process continues. Eventually something (change in circulation, vorticity, etc.) stops the forward movement and the shock starts to move aft and weaken. Again, the process is self-sustaining because U_{dif} increases, eddy viscosity gets larger, and the separated region decreases. The shock continues to move aft until the flow is fully attached on that surface (and the opposite surface shock is at its most forward location), and then the cycle repeats. There is nothing in the definition of F_{wake} to defeat this self-sustaining movement of the shock.

Modification to Turbulence Model

Flow in the separated region is characterized by a reversed flow region near the body. The elimination of all reference to the thickness of this separated region in the eddy-viscosity model does not seem appropriate. Therefore, a modification of the separated-flow/wake formulation [Eq. (2)] for F_{wake}, that includes a weighted function of the height of the reversed flow region, was developed. The resulting equation may be written as

$$F_{\text{wake}} = \frac{C_{\text{wk}} Y_{\max} U_{\text{dif}}^2}{F_{\max}} + (D_{\text{wk}} Y_0) \frac{C_{\text{wk}} Y_{\max} U_{\text{dif}}^2}{F_{\max}}$$

The last term represents additional dissipation that is proportional to the height (Y_0) of the reversed flow region.

The original equation for F_{wake} can be retained if a new, variable coefficient is defined as

$$C_{\text{wks}} = C_{\text{wk}}(D_{\text{wk}} Y_0 + 1)$$

C_{wks} is substituted for C_{wk} in the equation for F_{wake} and is computed at each x location. The value of C_{wks} changes in

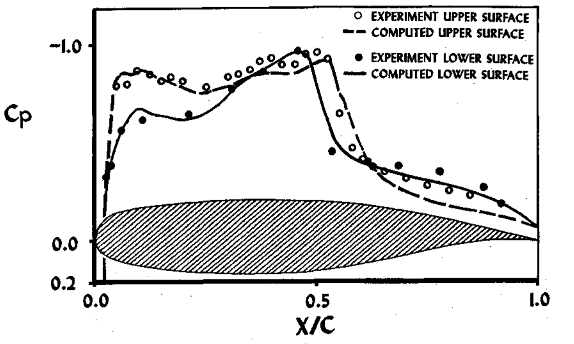

Fig. 3 Comparison with experimental data; $M = 0.807$, $\alpha = 0.37$, $Re = 12 \times 10^6$, and $D_{\text{wk}} = 50$.

direct proportion to the height of the reverse flow region, and provides the dissipation necessary to counteract the influence of U_{dif} changes.

The new parameter D_{wk} was evaluated to determine the magnitude required, and to assess its effect on the solution. Values from 20–120 were used. Figure 1 shows the effect on lift coefficient C_L of the different values for D_{wk}. As would be expected, the larger values of D_{wk} provide large eddy viscosity and produce results that approach solutions obtained using only the attached flow equation for F_{wake}. The smaller values of D_{wk} provide solutions that show signs of the oscillations observed with the original separated-flow/wake equation for F_{wake}. The lift approaches a minimum value at about $D_{\text{wk}} = 60$ (Fig. 1), and oscillates about this mean value at lower levels of D_{wk}. The pressure data is not strongly affected by the oscillations introduced by low values of D_{wk}. Figure 2 provides pressure data at four locations of an oscillation when $D_{\text{wk}} = 20$. There is very little effect on shock location or the flow upstream of the shock. Small differences exist at the foot of the shock and the nearby separated region. Figure 3 provides a comparison of computed and experimental results for $D_{\text{wk}} = 50$.

Conclusions

Stable numerical solutions of flows with turbulent boundary layers and shock-induced separation are very difficult to obtain. The limiting problem is associated with the closure model for the Reynolds-averaged Navier–Stokes equations. This Note provides a possible explanation of the process that allows oscillatory behavior in a Navier–Stokes solver when a Baldwin–Lomax algebraic eddy-viscosity model is employed. A modification to the Baldwin–Lomax model, that allowed a good solution for two-dimensional flow over a thick supercritical airfoil with strong shocks and massively separated flow, is described. The modified turbulence-model solution accurately predicted shock location on both surfaces. There are small discrepancies in the pressures on the upper surface ahead of the shock and in the separated region downstream of the shock.

This procedure is certainly not a cure for the closure problem, but it does provide some insight into the problem and allows stable solutions for some flow conditions that are not otherwise possible.

References

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